Team Test 2015 - KEY

The team test has 12 "free" questions, meaning they are all independent and stand alone. The last 3 questions, however, use the answers from the previous 12 questions as values in the problem, and are worth more points. In questions #13-15, a number referred to as [Px] is referring to the answer of the x^{th} question. For instance, [P11] is the answer to the 11^{th} question. Good luck!

1. (3) 15 years ago, Annie's grandmother was three times the age of Annie's father, who in turn was three times the age of Annie. Annie is 88 years younger than her grandmother. How old is Annie today?

Answer: 26

Solution: *From the information given, we can write 2 equations: A + 88 = G and G - 15 = 3(F - 15) = 9(A - 15) . We can substitute A+88 in for G, getting A + 73 = 9(A - 15) = 9A - 135 -> 73 = 8A - 135 -> 208 = 8A -> A = 26.*

2. (3) A 12"-diameter pizza is cut into 6 congruent slices while a 16"-diameter pizza is cut into 8 congruent slices. Quinn ate 3 slices of the 16" pizza while Andreas ate 3 slices of the 12" pizza. How many more square inches of pizza than Andreas did Quinn eat? This value can be expressed in the form $a\pi$; what is a?

Answer: 6

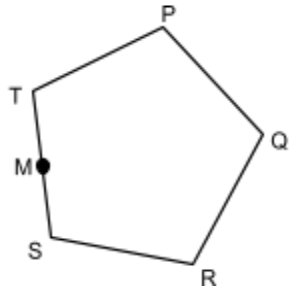
Solution: *A 12" pizza has an area of 36in^2, so 36of that is 18in^2. A 16" pizza has an area of 64in^2, so 38of that is 24in^2. 24 - 18 = 6, so the answer is 6.*

3. (3) Insanity affects 1% of the population. The Legallo-Ravichandran Insanity Test is 90% accurate. If the test declares you insane and they throw you in a straight-jacket, what is the probability that you're sane after all? This probability can be expressed as a fraction in the form $\frac{a}{b}$; concatenate ab.

Answer: 1112

Solution: *The probability that the you are insane is 0.90 x 0.01 = 0.009 . The probability that the test was wrong about your insanity is 0.99 x 0.10 = 0.099 . 0.0990.009 + 0.099=99108=1112, which concatenated would be 1112.*

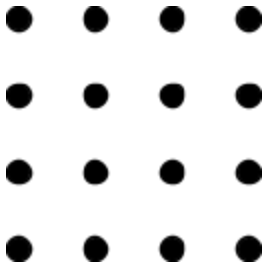
4. (4) M is the midpoint of ST in regular pentagon PQRST. What is the measure of <MQS?



Answer: 18

Solution: *Since the pentagon is regular, each of the interior angles is 180(5 - 2) / 5 = 108°. Triangle SQR is isosceles with <R = 108°, so <QSR = 36°. Therefore, <MSQ = 108 - 36 = 72°. Since <QMS = 90°, using triangle QSM we can determine that <MQS = 180 - 72 - 90 = 18°.*

5. (4) In the rectangular 4 by 4 grid below, horizontally and vertically adjacent dots are 1 unit apart. Using the grid points as endpoints, there are *n* segments, each with a different length. What is the median length of these *n* segments? This length can be expressed in the form $\sqrt{a}$; what is *a*?



Answer: 8

Solution: *There are segments of length 1, 2, and 3. Using diagonals, there are endpoints of length 2, 22, and 32. Also, there diagonals of length 5(1 by 2 triangle), 10(1 by 3 triangle), and13(2 by 3 triangle). In order from least to greatest, we have lengths*

6. (4) A soda company sells its product in either a 20-ounce bottle or a 12-ounce can. It charges $1.50 for the 20-ounce size. How much should they charge, in cents, for the can if they want the price per ounce to be 30% more than the price per ounce of the larger size?

Answer: 117

Solution: *The price per ounce of the bottle is 150 / 20 = 7.5 cents per ounce. Since they want to increase this by 30%, the price per ounce of the can is 7.5 x 1.3 = 9.75 cents per ounce. Therefore, the cost of the can is 12 x 9.75 = 117 cents.*

7. (4) A data set of ages lists the ages of a class of 20 eighth graders as integer values of either 13 or 14. The median age in the data set is 0.35 years less than the mean. How many 13-year-olds are in the class?

Answer: 13

Solution: *The mean age is equal to 13x+14y20, x + y = 20. This simplifies like so -> 14(x+y) - x20=14(20) - x20=14-x20. The mean is therefore less than 14. Since the median has to be either 13, 13.5, or 14, and the median is less than the mean, the median can't be 14. If the*

median was 13.5, then the average would also be 13.5, so the media

8. (4) The positive 3-digit integer K gives a perfect square when divided by 4 and gives a perfect cube when divided by 9. What is the value of K?

Answer: 576

Solution: *Given a system of equations, $K = 4x^2 = 9y^3$. 4 and 9 must be factors of K, so K is divisible by 36. Also, since 4 is a perfect square, $4x^2$ and by extension, K, must be perfect squares, and K is therefore divisible by 36=6. Of the numbers that have 3-digit squares, 12^2, 18^2, 24^2, and 30^2 fit. Remove the (3 x 3) from each of these to see which one contains a triplet. $12^2 = (3)^2 \times (2 \times 2)^2$. $18^2 = (3)^2 \times (2 \times 3)^2$. $24^2 = (3)^2 \times (2 \times 2 \times 2)^2$. $30^2 = (3)^2 \times (2 \times 5)^2$. 24^2 = 576 must be the value of K.*

9. (4) 5 couples are at a party. 4 people of the 10 are randomly selected to win a prize. What is the probability that both members of at least one couple win a prize? This probability can be expressed as a fraction in the form $\frac{a}{b}$; concatenate ab.

Answer: 1321

Solution: *The easiest way is to take the inverse of the probability - that is, find the probability that no couples would have both members selected. That probability would be* $\frac{1010896847}{} = 821$. Therefore, the probability that at both members of at least one couple are selected is 1-821 =1321. Concatenated, this would be 1321.

10. (5) A wall made of a certain number of rows consisting of cube-shaped bricks starts with a row of 23 bricks at the bottom. Each row has 4 fewer bricks than the row below it. Ishpreet noticed that the total number of bricks used in the wall is enough to create a rectangular brick floor that is one layer of bricks thick, 3 times as long as it is wide. How many rows of bricks does the wall have?

Answer: 5

Solution: *Since the length of the floor is 3 times the width, the number of bricks used is $3w^2$. This means that the number of bricks used in the wall has to be divisible by both 3 and a perfect square. The sequence of bricks is such that nth row has $a_n = 27 - 4n$, and the sum of the bricks is $s_n = n2(23 + 27 - 4n) = 25n - 2n^2$. n has to be between 1 and 6, inclusive. Obviously n = 1 won't work. After trying all of the other numbers, we find that n = 5 because s_n is both divisible by 3 and 25, a perfect square.*

11. (5) Two circles, one of radius 3 inches, the other of radius 5 inches, are tangent at point T. Two bugs start flying at the same time from point T, one flying around the larger circle at 3π inches per second while the other flies around the smaller circle at 2π inches per second. How many seconds from the time they start until their next meeting at point T?

Answer: 10

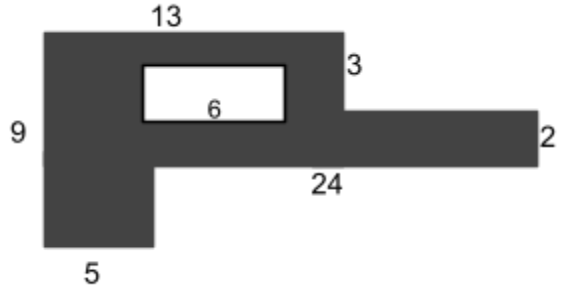
Solution: *The circumference of the larger circle is 10, so the first bug makes a full revolution every 103seconds. The circumference of the smaller circle is 4, so the second bug makes a full revolution every 2 seconds. We need to find the least common multiple of 103and 2 - this is*

easiest to do when we compare 103to 63. Ignoring the 3 in the denominator, the LCM of 6 and 10 is 30. Therefore, the time the bugs meet is 303=10seconds.

12. (5) The sum of five positive integers that form an arithmetic sequence is 65. Of all such possible sequences, what is the greatest possible fourth term?
Answer: 19
Solution: *Since an arithmetic sequence has terms which differ by the same amount, we can quantify this by stating x + (x + y) + (x + 2y) + (x + 3y) + (x + 4y) = 65 = 5x + 10y. We can reduce this to x + 2y = 13 , which is the same as the third term in the sequence. Since x and y both have to be positive integers, x has to be an odd number greater than or equal to 1. As x increases, y decreases, so the largest fourth term will be one with the smallest first term. If x = 1, y = 6 , so the fourth term is 19.*

13. (7) The quantity ([P9] + [P10] - 2) people are seated around a circular stadium. Each person is either a Patriot, who always tells the truth, or a Warrior, who always lies. If each person in the stadium says that the person to their right is a Warrior, how many Warriors are seated in the stadium?
Answer: 617
Solution: *If we start with a Patriot, the person to his right must be a Warrior. The next person must then be a Patriot, and so on. This means that there are an equal number of Patriots and Warriors, so 1234 / 2 = 617.*

14. (7) The area of the shaded region is ([P4] * [P5] -45) square units. All angles are right angles. What is the perimeter of the non-shaded region (the outlined one with the 6 in it)?



Answer: 18
Solution: *Since we know the area of the shaded region, we can find the area of the non-shaded region. Break the shape into 3 rectangles -> 5 x 4, 13 x 5, and 16 x 2. The area is 117, so the area of the non-shaded region is 117 - 99 = 18. Since the width of the non-shaded region is 6, the height is 18 / 6 = 3, so the perimeter is 2(3 + 6) = 18.*

15. (7) What is the value of the units digit of $3^{[P7]*2-2} + 6^{[P1]}$?
Answer: 7
Solution: *Since we are only looking for units digit values, we only have to multiply out to the units digit.*

3^1 -> 3
3^2 -> 9
3^3 -> 7
3^4 -> 1
3^5 -> 3
This pattern will repeat every four powers, so we just have to find which value corresponds to 3^{24}. Since 24 divides into 4 with no remainder, 3^{24} must have a units digit of 1. Now we must find the same for 6.
6^1 -> 6
6^2 -> 6
6^3 -> 6
This pattern is easy; 6 to any power has a units digit of 6. Therefore, 1 + 6 = 7.